MENIIT

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JEE MAIN-2022

COMPUTER BASED TEST (CBT)

DATE : 28-06-2022 (EVENING SHIFT) | TIME : (3.00 PM to 6.00 PM)

Duration 3 Hours | Max. Marks : 300

QUESTIONS & SOLUTIONS

PART : PHYSICS

1. Velocity (υ) and acceleration (a) in two systems of units 1 and 2 are related as $\upsilon_2 = \frac{n}{m^2}\upsilon_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constants. The relations for distance and time in two system respectively are:

(A) $\frac{n^3}{m^3}L_1 = L_2 \text{and} \frac{n^2}{m}T_1 = T_2$

(B) $L_1 = \frac{n^4}{m^2}L_2 \text{and} T_1 = \frac{n^2}{m}T_2$

(C) $L_1 = \frac{n^2}{m}L_2 \text{and} T_1 = \frac{n^4}{m^2}T_2$

(D) $\frac{n^2}{m}L_1 = L_2 \text{and} \frac{n^4}{m^2}T_1 = T2$

Ans. (A)

Sol. $\frac{v^1}{v^2} = \frac{a_1 t_1}{a_2 t_2}$

$\frac{v^1}{v^2} = \frac{m_2}{n}$

$\frac{a^1}{a^2} = mn$

$\frac{m^2}{\eta} = mn\frac{t_1}{t_2}$

$T_2 = \frac{\eta^2}{m}T_1$

2. A ball is spun with angular acceleration α = 6t^2 – 2t where t is in second and α is in rads^{-2}. At t = 0, the ball has angular velocity of 10 rads^{-1} and angular position of 4 red. The most appropriate expression for the angular position of the ball is:

(A) $\frac{3}{2}t^4 - t^2 + 10t$

(B) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(C) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(D) $2t^4 - \frac{t^3}{2} + 5t + 4$

Ans. (B)

Sol. $\alpha = 6t^2 - 2t$

$$\frac{d\omega}{dt} = 6t^2 - 2t$$

$$\int_{10}^{\omega} d\omega = \int_{0}^{t} (6t^2 - 2t)dt$$

$$\omega - 10 = 2t^3 - t^2$$

$$\frac{d\theta}{dt} = 10 + 2t^3 - t^1$$

$$\int_{4}^{\theta} d\theta = \int_{0}^{t} (10 - 2t^3 - t^2)dt$$

$$\theta - 4 = 10t + \frac{t^4}{2} - \frac{t^3}{3}$$

3. A block of mass 2 kg moving on a horizontal surface with speed of ms^{-1} enters a rough surface ranging from x = 0.5 m to x = 1.5 m. The retarding force in this range of rough surface is related to distance by F = –kx where k = 12 Nm^{-1}. The speed of the block as it just crosses the rough surface will be:

(A) zero

(B) 1.5 ms^{-1}

(C) 2.0 ms^{-1}

(D) 2.5 ms^{-1}

Ans. (C)

Sol. F = – kx

$K = 12\ Nm^{-1}$

a = 6x

$$\int_{4}^{v} vdv = \int_{0.5}^{1.5} -3xdx$$

$$\frac{v^2 - 16}{2} = \frac{6}{2}[2.25 - 0.25]$$

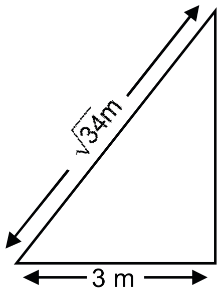
$$V^2 = -12 + 16$$

$$V = \sqrt{4}$$

V = 2m/s

4. A $\sqrt{34}$ m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor m away from the wall as shown in the figure. If F_f and F_w are the reaction forces of the floor and the wall, then ratio of F_w / F_f will be:

(Use g = 10 m/s^2.)



(A) $\frac{6}{\sqrt{110}}$

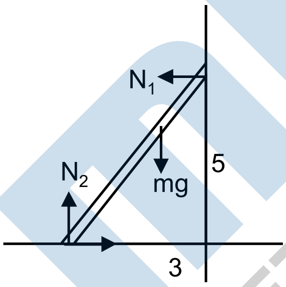
(B) $\frac{3}{\sqrt{113}}$

(C) $\frac{3}{\sqrt{109}}$

(D) $\frac{2}{\sqrt{109}}$

Ans. (D)

Sol.



$N_1 = f_2$, $N_2 = mg$

$$N_1 \times 5 = mg \times \frac{3}{2} \Rightarrow N_1 = \frac{3}{10} mg$$

$$R_1 = N_1 = \frac{3}{10} mg, R_2 = \sqrt{N_2^2 + f_2^2} = \frac{\sqrt{109}}{10} mg$$

$$\frac{R_1}{R_2} = \frac{3}{\sqrt{109}} \frac{F_w}{F_f} \frac{3}{\sqrt{109}}$$

5. Water falls from a 40 m high dam at the rate of 9×10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical. Using this hydroelectric energy number of 100 W lamps, that can be lit, is :

(Take $g = 10\ ms^{-2}$)

(A) 25

(B) 50

(C) 100

(D) 18

Ans. (B)

Sol. $\frac{40 \times 9 \times 10^4}{1hr} g \times \frac{50}{100} = \frac{40 \times 9 \times 10^4}{3600} \times 10 \times \frac{50}{100} = 100N$

N = 50

6. Two objects of equal masses placed at certain distance from each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be :

(A) $\frac{2}{9}F$

(B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$

(D) F

Ans. (C)

Sol. $F \frac{Gmm}{d^2}$

$$F' \frac{G\frac{2m}{3} \times \frac{4}{3}m}{d^2} = \frac{8}{9}\frac{Gmm}{d^2}$$

$$\frac{F'}{F} = \frac{8}{9}$$

$$F' = \frac{8}{9}F$$

7. A water drop of radius 1 μm falls in a situation where the effect of buoyant force is negligible. Co-efficient of viscosity of air is 1.8×10^{-15} Nsm^{-2} and its density is negligible as compared to that of water 10^6 gm^{-3}. Terminal velocity of the water drop is:

(Take acceleration due to gravity = $10\ ms^{-2}$)

(A) $145.4 \times 10^{-6}\ ms^{-1}$

(B) $118.0 \times 10^{-6}\ ms^{-1}$

(C) $132.6 \times 10^{-6}\ ms^{-1}$

(D) $123.4 \times 10^{-6}\ ms^{-1}$

Ans. (D)

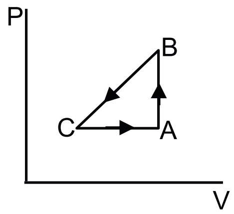
Sol. $\frac{4}{3}\pi r^3 \rho g = 6\pi n r V$

$\frac{4}{3 \times 6} r^2 \frac{\rho g}{n} = \upsilon$

$\frac{4}{3} \times \frac{10^{-12} \times 10^3 \times 10}{1.8 \times 10^{-5} \times 6}$

$v = 123.4 \times 10^{-6}$ m/s

8. A sample of an ideal is taken through the cyclic process ABCA as shown in figure. It absorb, 40 J of heat during the part AB, no heat during BC and rejects 60 J of heat during CA. A work of 50 J is done on the gas during the part BC. The internal energy of the gas at A is 1560 J. The workdone by the gas during the part t CA is:



(A) 20 J

(B) 30 J

(C) – 30 J

(D) –60 J

Ans. (B)

Sol. For cycle process

Total heat = $W_{total} + \Delta v$

$-60 + 40 + 0 = W_{CA} + W_{AB} + W_{BC}$

$-20 = W_{CA} + 0 + 30$

$W_{CA} = -50.$

9. What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxgen?

(A) The velocity of atomic oxygen remains same

(B) The velocity of atomic oxygen doubles

(C) The velocity of atomic oxygen becomes half

(D) The velocity of atomic oxygen becomes four times

Ans. (B)

10. Two point charges A and B magnitude + 8 × 10^{-6} C and – 8 × 10^{-6} C respectively are placed at a distance d apart. The electric field at the middle point O between the charges is 6.4 × 10^{4} NC^{-1}. The distance 'd' between the point charges A and B is :

(A) 2.0 m

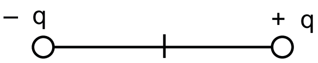
(B) 3.0 m

(C) 1.0 m

(D) 4.0 m

Ans. (B)

Sol.



E at mid point

$$E = \frac{2kp}{d^2}$$

$$6.4 \times 10^4 = \frac{8kp}{d^2}$$

$$d^2 = \frac{8 \times k \times 8 \times 10^{-6}}{6.4 \times 10^4} = \frac{8 \times 9 \times 10^9 \times 8 \times 10^{-6}}{6.4 \times 10^4} = 3m$$

11. Resistance of the wire is measured as 2Ω and 3Ω at 10°C and 30°C respectively. Temperature co-efficient of resistance of the material of the wire is:

(A) 0.033 $°C^{-1}$

(B) –0.033 $°C^{-1}$

(C) 0.011 $°C^{-1}$

(D) 0.055d $°C^{-1}$

Ans. (A)

Sol. $R = R_0(1 + \alpha \Delta T)$
$2 = R_0(1 + 10\alpha)$
$3 = R_0(1 + 30\alpha)$
$1 = 30\alpha$

$$\alpha = \frac{1}{30} = 0.033$$

12. The space inside a straight current carrying solenoid is filled with a magnetic material having magnetic susceptibility equal to 1.2 × 10^{-5}. What is fractional increase in the magnetic field inside solenoid with respect to air as medium inside the solenoid?

(A) 1.2×10^{-5}

(B) 1.2×10^{-3}

(C) 1.8×10^{-3}

(D) 2.4×10^{-5}

Ans. (A)

Sol. $\chi = 1.2 \times 10^{-5}$

$\mu_r = \chi - 1$

$B = {}_{\mu}ni$

$= {}_{\mu r \mu 0}ni$

13. Two parallel, long wires are kept 0.20 m part in vacuum, each carrying current of x A in the same direction. If the force of attraction per meter of each wire is 2×10^{-6} N, then the value of x is approximately :

(A) 1

(B) 2.4

(C) 1.4

(D) 2

Ans. (C)

Sol. $\frac{F}{\ell} = \frac{\mu_0 ii}{2\pi d}$

$$2 \times 10^{-6} = \frac{4\pi^2 \times 10^{-7} i^2}{2\pi \times 0.2}$$

$i^2 = \sqrt{2} = 1.4$

14. A coil is placed in a time varying magnetic field. If the number of turns in the coil were to be halved and the radius of wire doubled, the electrical power dissipated due to the current induced in the coil would be:

(Assume the coil to be short circuited.)

(A) Halved

(B) Quadrupled

(C) The same

(D) Doubled

Ans. (D)

Sol. Resistance of coil remains same if number of turn becomes half and radius is doubled.

$$E = \frac{Nd\phi}{dt}$$

$$= -\frac{NAdb}{dt}$$

$$P = \frac{e^2}{R}$$

$P \propto e^2 \propto N^2A^2 \propto N^2r^4$

$(1/2)^2 (2)^4 = 2^2$

15. An EM wave propagating in x-direction has a wavelength of 8 mm. The electric field vibrating y-direction has maximum magnitude of 60 Vm^{-1}. Choose the correct equation for electric and magnetic fields if the EM wave is propagating in vacuum:

(A) $E_y = 60\sin\left[\frac{\pi}{4}\times 10^3\left(x-3\times 10^8 t\right)\right]\hat{j}vm^{-1}$

$B_z = 2\sin\left[\frac{\pi}{4}\times 10^3\left(x-3\times 10^8 t\right)\right]\hat{k}T$

(B) $E_y = 60\sin\left[\frac{\pi}{4}\times 10^3\left(x-3\times 10^8 t\right)\right]\hat{j}vm^{-1}$

$B_z = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^3\left(x-3\times 10^8 t\right)\right]\hat{k}T$

(C) $E_y = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^3\left(x-3\times 10^8 t\right)\right]\hat{j}vm^{-1}$

$B_z = 60\sin\left[\frac{\pi}{4}\times 10^3\left(x-3\times 10^8 t\right)\right]\hat{k}T$

(D) $E_y = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^4\left(x-4\times 10^8 t\right)\right]\hat{j}vm^{-1}$

$B_z = 60\sin\left[\frac{\pi}{4}\times 10^4\left(x-4\times 10^8 t\right)\right]\hat{k}T$

Ans. (B)

16. In young's double slit experiment performed using a monochromatic light of wavelength λ, when a glass plate ($\mu = 1.5$) of thickness $x\lambda$ is the path of the one of the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:

(A) 3

(B) 2

(C) 1.5

(D) 0.5

Ans. (B)

Sol. $\Delta x = (\mu - 1)t$

$= (1.5 - 1)\, x\lambda = n\lambda \qquad n = 1$

$x\lambda = \frac{\lambda}{0.5}$

$x = 2$

17. Let K_1 and K_2 be the maximum kinetic energies of photo-electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2, respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then:

(A) $K_1 > \frac{K_2}{3}$

(B) $K_1 < \frac{K_2}{3}$

(C) $K_1 = \frac{K_2}{3}$

(D) $K_1 = \frac{K_1}{3}$

Ans. (B)

Sol. $K_1 = \frac{hc}{\lambda_1} - \phi$

$K_2 = \frac{hc}{\lambda_2} - \phi$

$$\frac{K_1}{K_2} = \frac{\frac{hc}{3\lambda_2} - \phi}{\frac{hc}{\lambda_2} - \phi}$$

$K_1 < \frac{K_2}{3}$

18. Following statements related to radioactivity are given below:

(A) Radioactivity is a random and spontaneous process and is dependent on physical and chemical conditions.

(B) The number of un-decayed nuclei in the radioactive sample decays exponentially with time.

(C) Slope of the graph of $\log_e$ (no. of undecayed nuclei) Vs. time represents the reciprocal of mean life time (τ).

(D) Products of decay constant (λ) and half-life time ($T_{1/2}$) is not constant.

Choose the **most appropriate** answer from the options given below:

(A) (A) and (B) only

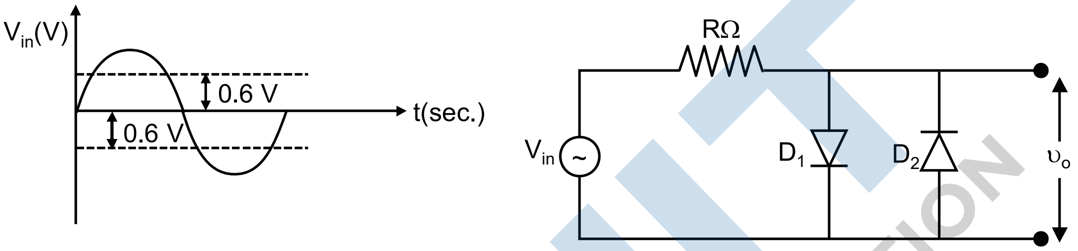
(B) (B) and (D) only

(C) (B) and (C) only

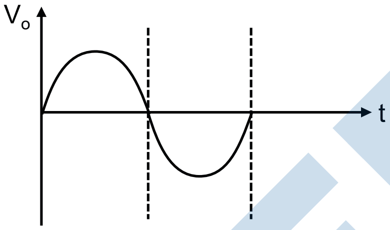
(D) (C) and (D) only

Ans. (C)

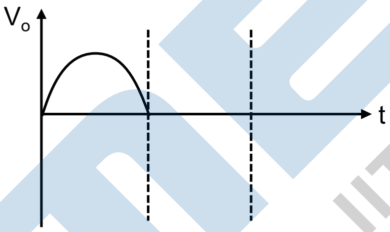
19. In the given circuit the input voltage V_{in} is shown is figure. The cut-in voltage of p-n junction diode (D_1 or D_2) is 6.0 V. Which of the following voltage (V_o) wavefrom across the diode is correct ?



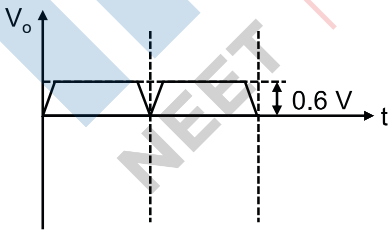
(A)



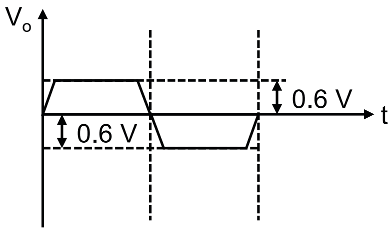
(B)



(C)



(D)



Ans. (D)

20. Amplitude modulated wave is represented by

$V_{AM} = 10\,[1 + 0.4 \cos(2\pi \times 10^4 t)] \cos(2\pi \times 10^7 t)$. The total bandwidth of the amplitude modulated wave is :

(A) 10 kHz

(B) 20 MHz

(C) 20 kHz

(D) 10 MHz

Ans. (C)

Sol. $f = \frac{\omega}{2\pi}$

Band width = 2f

21. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.22 mm, 1.23 mm, 1.19 mm and 1.20 mm. The percentage error is $\frac{x}{121}\%$. The value of x is ________.

Ans. (150)

Sol. $X_{avg} = \frac{1.19 + 1.20 + 1.22 + 1.23}{4}$

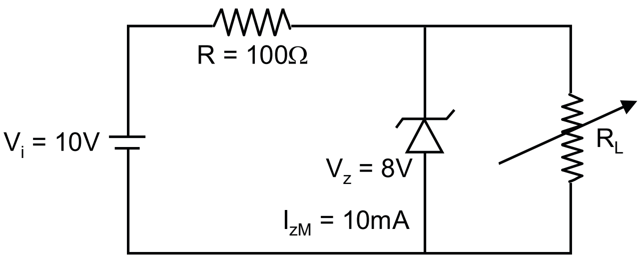
$$\Delta x = \frac{0.02 + 0.01 + 0.01 + 0.02}{4} = \frac{0.06}{4}$$

$$\Delta x = \frac{\frac{0.003}{2}}{1.21} \times 100$$

$$\Delta x = \frac{150}{121}$$

X = 150

22. A zener of breakdown voltage $V_Z = 8$ V and maximum zener current, $I_{ZM} = 10$ mA is subjected to input voltage $V_i = 10$V with series resistance R = 100 Ω. In the given circuit R_L represents the variable load resistance. The ratio of maximum and value of R_L is __________.



Ans. (2)

Sol. $R_L = \frac{8}{10} = 0.8$

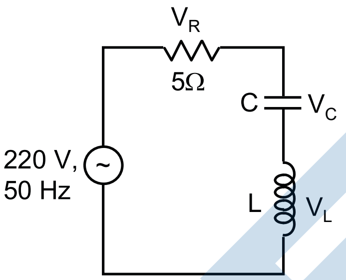
$R_{max} = \frac{8}{20}$

$\frac{8}{10} \times \frac{20}{8} = 2$

23. In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index 7/5, is $\frac{1}{\alpha}$. The value of α is _______________.

Ans. (4)

24. In the given circuit, the magnitude of V_L and V_C are twice that of V_R. Given that f = 50 Hz, the inductance of the coil is $\frac{1}{K\pi}$mH. The value of K is ______________.



Ans. JEE main answer is zero and zigyan answer is $\frac{1}{100}$

Sol. $v = \sqrt{v_2^2 + (v_L + v_C)^2}$

$v_S = v_C = v_R$

$v_S = v_R = 220$ V

$I_{rms} = \frac{220}{5} = 44A$

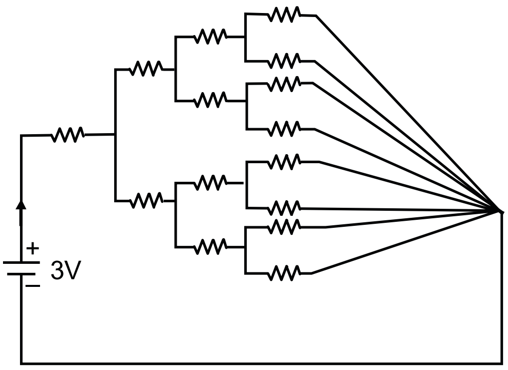
$X_L = \frac{440}{44} = 10\Omega$

$L = \frac{10}{100\pi} = \frac{1}{10\pi}Hz$

$\frac{1}{K\pi} \times 10^3 = \frac{1}{10\pi}$

$K = \frac{1}{100}$

25. All resistance in figure are 1 Ω each. The value of current 'I' is $\frac{a}{5}A$. The value of a is ____________.

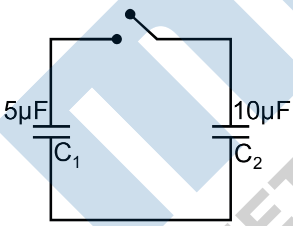


Ans. (8)

Sol. $R_{eq} = \frac{15}{8}$

$i = \frac{3}{\frac{15}{8}} = \frac{8}{5}A$

26. A capacitor C_1 of capacitance 5 μF is charged to a potential of 30 V using a battery. The battery is that removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance 10 μF as shown in figure. When the switch closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is __________ μC.



Ans. (100)

Sol. $v = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$= \frac{5 \times 30 + 0}{5 + 10} = 10$

$Q_2 = C_2V = 10 \times 10 = 100\mu C$

27. A tunning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is __________cm.

(Velocity of sound in air is 340 ms^{-1})

Ans. (50)

28. A liquid of density 750 kgm^{-3} flows smoothly through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.2 \times 10^{-2}\ m^2$ to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow sections of the pipe is 4500 Pa. The rate of flow of liquid is __________ $\times 10^{-3}\ m^3s^{-1}$.

Ans. (24)

Sol. $P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2}$

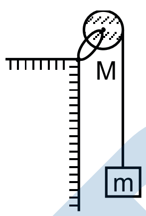
$P_1 - P_2 = \rho\left(\frac{v_2^2 - v_1^2}{2}\right)$

$4500 = 750\left(\frac{3v^2}{2}\right)$

V = 2

29. A uniform disc with mass M = 4 kg and radius R = 10 cm is mounted on a fixed horizontal axle as shown in figure. A block with mass m = 2 kg hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is _________ N.

(Take g = 10 ms^{-2})



Ans. (10)

Sol. $\tau = I\alpha$

$= \frac{4r^2}{2}\alpha$

$\alpha = \frac{T}{2r} = \frac{T}{2 \times 0.1} 5T$

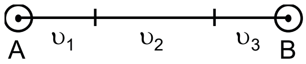
$2g - T = 2a = 2 \times 0.1 \times \alpha$

$20 - T = 0.2 \times 5T$

$20 = 2T$

$T = 10N$

30. A car covers AB distance with first one-third at velocity υ_1 ms^{-1}, second one-third at υ_2 ms^{-1} and last one-third at υ_3 ms^{-1}. If $\upsilon_3 = 3\upsilon_1$, $\upsilon_2 = 2\upsilon_1 = 11$ ms^{-1} then the average velocity of the car is ________ms^{-1}.



Ans. (18)

Sol. $V_{avg} = \dfrac{3d}{\dfrac{d}{11} + \dfrac{d}{22} + \dfrac{d}{33}} = \dfrac{3}{\dfrac{6+3+2}{66}} = 18m/s$

PART : CHEMISTRY

1. Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The, molecular formula of the compound is,

Given : Atomic masses of C, H and N are 12, 1 and 14 amu respectively.

The molar mass of the compound A is 162 g mol^{-1}.

(A) $C_4H_6N_2$

(B) C_2H_3N

(C) C_5H_7N

(D) $C_{10}H_{14}N_2$

Ans. (D)

Sol. GMM of $C_{10}H_{14}N_2$ $\Rightarrow$ 120 + 14 + 28

$\Rightarrow$ 162

2. Consider the following statements:

(A) The principal quantum number 'n' is positive integer with values of 'n' = 1, 2, 3, ...

(B) The azimuthal quantum number 'l' for a given 'n' (principal quantum number) can have values as 'l' = 0, 1, 2, ... n

(C) Magnetic orbital quantum number 'm_l' for a particular 'l' (azimuthal quantum number) has (2l + 1) values.

(D) ± 1/2 are two possible orientations of electron spin.

(E) For l = 5, there will be a total of 9 orbital

Which of the above statements are correct?

(A) (A), (B) and (C)

(B) (A), (C), (D) and (E)

(C) (A), (C) and (D)

(D) (A), (B), (C) and (D)

Ans. (C)

Sol. Value of ℓ for a given n^{th} orbit is equal to 0, 1, 2, ... (n – 1)

For ℓ = 5 total number of orbital is (2 ℓ + 1) = 11

3. In the structure of SF_4, the lone pair of electrons on S is in.

(A) equatorial position and three are two lone pair – bond pair repulsions at 90°.

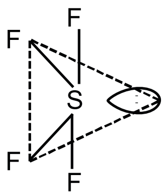
(B) equatorial position and three are three lone pair – bond pair repulsions at 90°.

(C) axial position and three are three lone pair – bond pair repulsions at 90°.

(D) axil position and three are two lone pair – bond pair repulsions at 90°.

Ans. (A)

Sol.



Lone pair at equatorial position with 2 lone pair – bond pair repulsion at 90°

4. A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH4.

The ratio of $\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$ required to make buffer is _________.

Given: $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$

(A) 0.03

(B) 0.13

(C) 0.23

(D) 0.33

Ans. (B)

Sol. $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$

$pK_a = 5 - \log 1.3$

$$pH = pK_a + \log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$4 = 5 - \log 1.3 + \log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$\log 1.3 - 1 = \log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$0.114 - 1 = \log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = \text{antilog}(-0.886) = 0.3$$

5. Match **List – I** with **List – II**:

List – I	List – II
(A) negatively charged sol	(I) $Fe_2O_3 \cdot xH_2O$
(B) macromolecular colloid	(II) CdS sol
(C) Positively charged sol	(III) Starch
(D) Cheese	(IV) a gel

Choose the correct answer from the options given below:

(A) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(B) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

(C) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) – (I), (B) – (III), (C) – (II), (D) – (IV)

Ans. (C)

Sol.

Positively charged sol $\Rightarrow Fe_2O_3 \cdot xH_2O$

Macro molecular colloid $\Rightarrow$ Starch

Negative Colloid $\Rightarrow$ CdS sol

Gel $\Rightarrow$ Cheese

6. Match **List – I** with **List – II**:

List – I **List – II**

(A) Cl_2O_7	(I) Amphoteric
(B) Na_2O	(II) Basic
(C) Al_2O_3	(III) Neutral
(D) N_2O	(IV) Acidic

Choose the correct answer from the options given below:

(A) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(B) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

(C) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

(D) (A) – (I), (B) – (I), (C) – (III), (D) – (IV)

Ans. (B)

Sol. Cl_2O_7 - Acidic

Na_2O - Basic

Al_2O_3 - Amphoteric

N_2O - Natural

7. In the metallurgical extraction of copper, following reaction is used:

$FeO + SiO_2 \rightarrow FeSiO_3$

FeO and $FeSiO_3$ respectively are.

(A) gangue

(B) flux and slag.

(C) slag and flux.

(D) gangue and slag.

Ans. (D)

Sol. FeO + $SiO_2 \longrightarrow FeSiO_3$

Gangue Fiux slag

8. Hydrogen has three isotopes: protium (^1H), deuterium (^2H or D) and tritium (^3H or T). They have nearly same chemical properties but different physical properties. They differ in

(A) number of protons

(B) atomic number.

(C) electronic configuration.

(D) atomic mass.

Ans. (D)

Sol. Isotopes have same proton & electron but different in number of neutron and mass number.

9. Among the following, basic oxide is :

(A) SO_3

(B) SiO_2

(C) CaO

(D) Al_2O_3

Ans. (C)

Sol. Acidic $\Rightarrow$ SOi_2, SO_2

Amphoteric $\Rightarrow$ Al_2O_3

Basic $\Rightarrow$ CaO

10. Among the given oxides of nitrogen; N_2O, N_2O_3, N_2O_4 and N_2O_5, the number of compound/ (S) having N – N bond is:

(A) 1

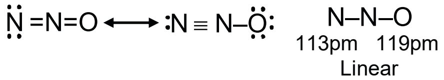
(B) 2

(C) 3

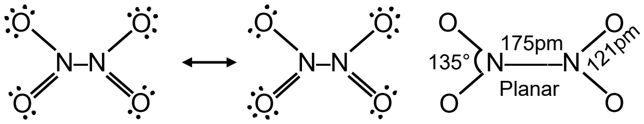
(D) 4

Ans. (C)

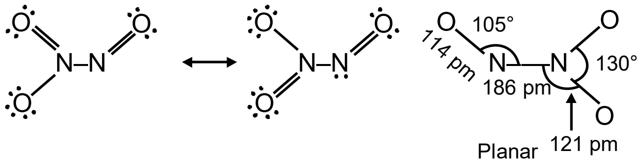
Sol. N_2O



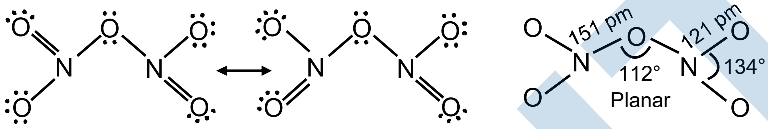
N_2O_4



N_2O_3



N_2O_5



11. Which of the following oxoacids of sulphur contains "S" in two different oxidation states?

(A) $H_2S_2O_3$

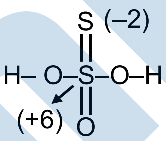
(B) $H_2S_2O_6$

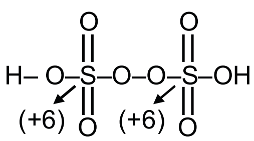
(C) $H_2S_2O_7$

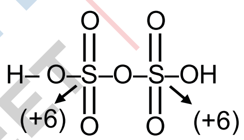
(D) $H_2S_2O_8$

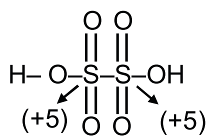
Ans. (A)

Sol.

$H_2S_2O_3 \Rightarrow$ 

$H_2S_2O_2 \Rightarrow$ 

$H_2S_2O_7 \Rightarrow$ 

$H_2S_2O_6 \Rightarrow$ 

12. Correct statement about photo-chemical smog is :

(A) It occurs in humid climate.

(B) It is a mixture of smoke, fog and SO_2.

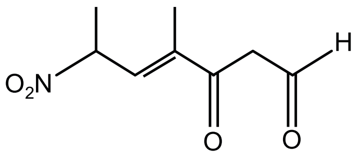
(C) It is reducing smog.

(D) It result from reaction of unsaturated hydrocarbons.

Ans. (D)

Sol. It is fact.

13. The correct IUPAC name of the following compound is :



(A) 4-methyl-2-nitro-5-oxohept-3-enal

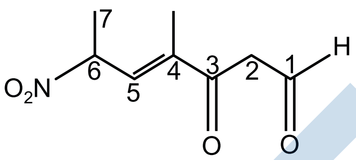
(B) 4-methyl-5-oxo-2-nitrohept-3-enal

(C) 4-methyl-6-nitro-3-oxohept-4-enal

(D) 6-formyl-4-methyl-2-nitrohex-3-enal

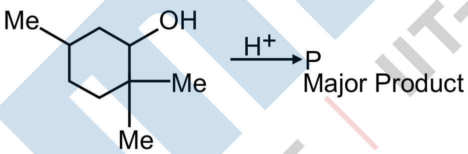
Ans. (C)

Sol.

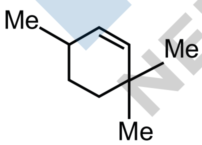


14. The major product (P) of the given reaction is

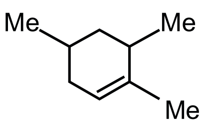
(where, Me is $-CH_3$)

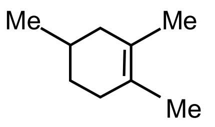


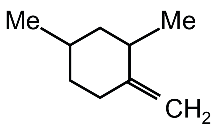
(A)



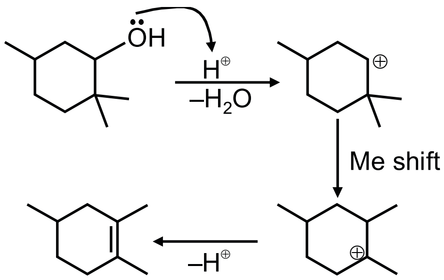
(B)



(C) 

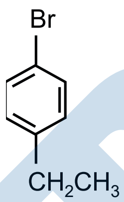
(D) 

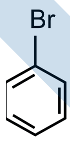
Ans. (C)

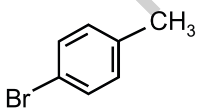
Sol. 

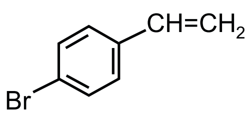
15. $A \xrightarrow[\text{(ii) } CH^{-} \quad \text{(iii) } H_2O/H^{+}]{\text{(i) } CH_2,\Delta}$ 4-Bromophenyl acetic acid.

In the above reaction 'A' is

(A) 

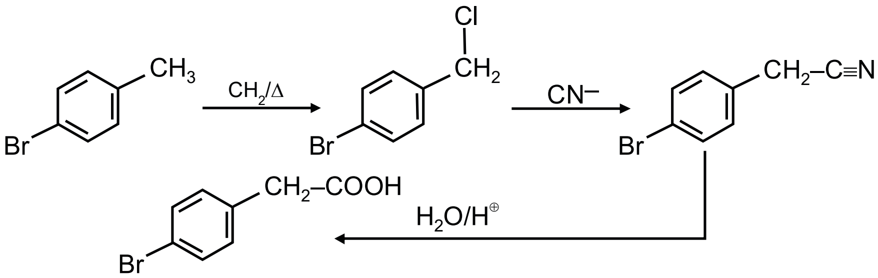
(B) 

(C) 

(D) 

Ans. (C)

Sol.



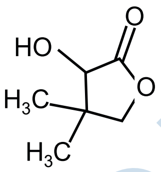
4-Bromophyenyl acetic acid

16. Isobutyraldehyde on rection with formaldehyde and K_2CO_3 give compounds 'A'. Compound 'A' reacts with KCN and yields compounds 'B', which on hydrolysis gives a stable compounds 'C'. The compound 'C' is

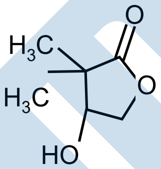
(A) $HO-CH_2-C(CH_3)(CH_3)-CH(OH)-COOH$

(B) $HO-CH_2CH_2-CH(CH_3)-CH(OH)-COOH$

(C)

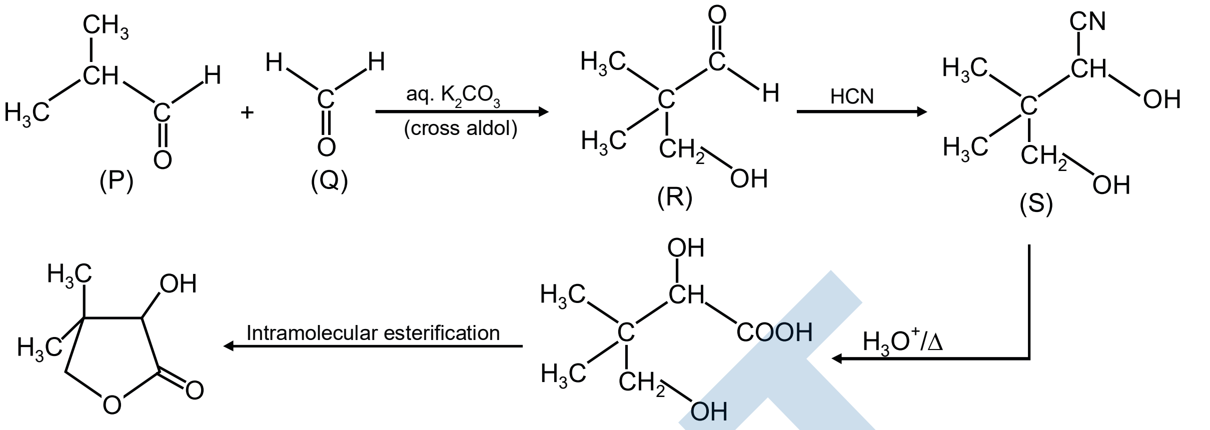


(D)

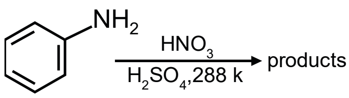


Ans. (C)

Sol.



17. With respect to the following reaction, consider the given statement:



(A) o-Nitroaniline and p-nitroaniline are the predominant are the products.

(B) p-Nitroaniline and m-nitroaniline are the predominant are the products.

(C) HNO_3 acts as an acid.

(D) H_2SO_4 acts as an acid.

Choose the correct option.

(A) (A) and (C) are correct statements.

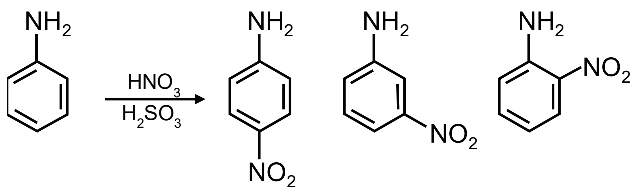
(B) (A) and (D) are correct statements.

(C) (B) and (D) are correct statements.

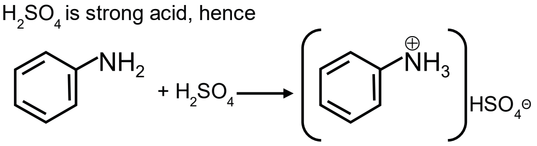
(D) (B) and (C) are correct statements.

Ans. (C)

Sol.



H_2SO_4 is strong acid, hence



18. Given below are two statements, one is Assertion (A) and other is Reason (R).

Assertion (A): Natural rubber is a liner polymer of isoprene called cis-polyisoprene with elastic properties.

Reason (R): The cis-polyisoprene molecules consist of various chains held together by strong polar interactions with coiled structure.

In the light of the above statements, choose the correct one from the options given below:

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

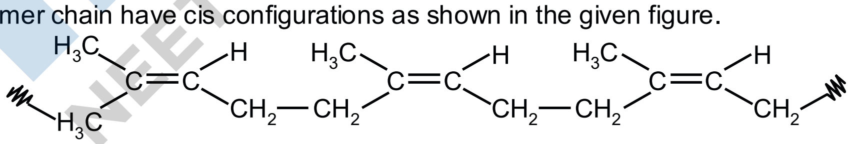
(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Ans. (C)

Sol. Natural rubber

Natural rubber is a polymer of isoprene, and obtained from natural source-latex tree. In natural rubber, isoprene units are joined together in head-to-tail fashion and all double bonds in the polymer chain have cis configurations as shown in the given figure.



Cis-1, 4-polysoprene

(Natural rubber)

The polymer contains cis repeating units and has a molecular weight ranging from 100,000 up to 1,000,000. The cis arrangement of the double bonds in natural rubber prevents the rubber molecules from fitting into an ordered structure. Thus, rubber is an amorphous polymer. Because of the random coiling of its polymer chains, rubber stretches easily. When stretched, the rubber molecules are forced into a higher energy state. when the tension is released, rubber snaps back to its original random coiled state but it is nonpolar therefore statement-II is incorrect.

19. When sugar 'X' is boiled with dilute H_2SO_4 in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with HNO_3 yields saccharic acid where as 'B' is levorotatory. The compound 'X' is:

(A) Maltose

(B) Sucrose

(C) Lactose

(D) Starch

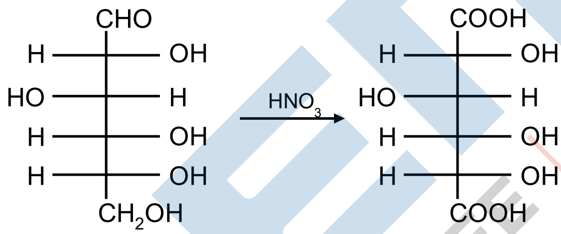
Ans. (B)

Sol. $X \xrightarrow{H_3O^+} A + B$ (levorotatory)

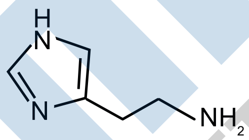
$A \xrightarrow{HNO_3}$ (Sacharic acid)

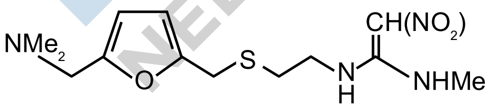
$C_{12}H_{22}O_{11} + H_2O \xrightarrow{HCl} C_6H_{12}O_6$ (D - glucose) $+ C_6H_{12}O_6$ (D - fructose)

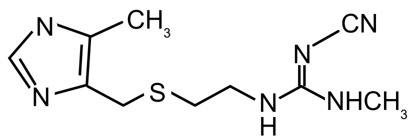
$[\alpha] = +66.6°$ (X) $\quad [\alpha] = +52.7°$ (A) $\quad [\alpha] = -92.2°$ (B)



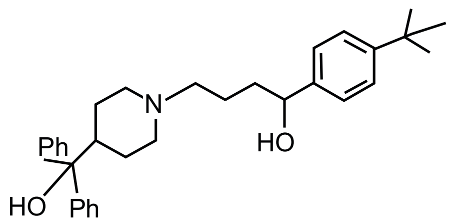
20. The drug tegamet is:

(A)


(B)


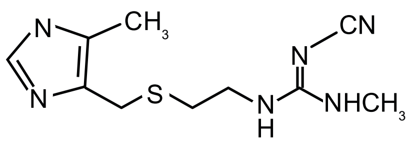
(C)


(D)



Ans. (C)

Sol.



21. 100 g of an ideal gas is kept in a cylinder of 416 L volume at 27°C under 1.5 bar pressure. The molar mass of the gas is __________ g mol^{-1}.

(Given : R = 0.083 l bar K^{-1} mol^{-1})

Ans. (4)

Sol. PV = nRT

$$1.5 \times 416 = \frac{100}{M.wt} \times 0.083 \times 300$$

M.Wt. = 3.99 = 4 g/mol

22. For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_C H^1$ = – 601.7 kJ mol^{-1}, the magnitude of change in internal energy for the reaction is _________kJ.

(Given : R = 8.3 J K^{-1} mol^{-1})

Ans. (600)

Sol. $Mg_{(S)} + \frac{1}{2}O_{2(G)} \rightarrow MgO(S)\Delta H_c^{\circ} = -601.70 KJ/Mole$

$\Delta H° = \Delta U + \Delta ngRT$

$$-601.70 = \Delta U + \left[\left(-\frac{1}{2}\right) \times 8.3 \times 300\right] \times 10^{-3}\ 3.$$

$-601.7 = \Delta U - 1.245$

$\Delta U = -599.455$ KJ

$| \Delta U | = 599.455 \text{ KJ} \approx 600$

23. 2.5 g of protein containing only glycine ($C_2H_5NO_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be 5.03×10^{-3} bar. The total number of glycine units present in the protein is ____________.

(Given : R = 0.083 L bar $K^{-1}mol^{-1}$)

Ans. (330)

Sol. p = CRT

$$5.03 \times 10^{-3} = \left[\frac{2.5 \times 1000}{M.wt \times 500}\right] \times 0.083 \times 300$$

M.Wt = 24.752ss' 10^3 gram = 24752gram

Molar mass of glycine (NH_2CH_2COOH) = 75 g/Mol.

No of glycine unit in protein = $\frac{24752}{75} = 330$

24. For the given reactions

$Sn^{2+} + 2e^- \rightarrow Sn$

$Sn^{4+} + 4e^- \rightarrow Sn$

the electrode potentials are ; $E^o_{Sn^{2+}/Sn} = -0.140V \text{ and } E^o_{Sn^{4+}/Sn} = -0.010V$. The magnitude of standard electrode potential for Sn^{4+}/Sn^{2+} i.e $E^o_{Sn^{4+}/Sn^{2+}}$ is ___________ × 10^{-2} V.

Ans. (16)

Sol. (i) $Sn^{2+} + 2e^- \rightarrow Sn \quad E^0_1 = -0.14V$

$\Delta G^0_1 = -2F(-0.14V)$

(ii) $Sn^{4+} + 4e^- \rightarrow Sn \quad E^0_2 = +0.010V$

$\Delta G^0_2 = -4F(+0.010)$

Target $Sn^{4+} + 2e^- \rightarrow Sn^{2+} \quad E^0_3 = ?$

$\Delta G^0_3 = -2F\left[E^0_3\right]$

Target Eq. = Eq. ii – Eq. i

$$-2F\left(E_3^0\right) = -4F(0.010) - \left(-2F(-0.14)\right)$$

$$E_3^0 = \frac{4\times 0.010 + 2\times 0.14}{2} = 0.16V = 16\times 10^{-2}V$$

25. A radioactive element has a half life of 200 days. The percentage of original activity remaining after 83 days is __________.

(Given : antilog 0.125 = 1.333, antilog 0.693 = 4.93)

Ans. (75)

Sol. $\text{Activity} = \frac{-d}{dt}[N] = \lambda[N]$

% activity remaining after 83 day

$$\left(\frac{N}{N_0}\right)100 = e^{-\lambda t} = \left[e^{-\frac{\ell n2}{200}\times 83}\right]100$$

$$\left(\frac{N}{N_0}\right)100 == \left[e^{-0.287}\right]\times 100 = 75$$

26. $[Fe(CN)_6]^{4-}$

$[Fe(CN)_6]^{3-}$

$[Ti(CN)_6]^{3-}$

$[Ni(CN)_4]^{2-}$

$[Co(CN)_6]^{3-}$

Among the given complexes, number of paramagnetic complexes is ____________.

Ans. (2)

Sol.

Complex	Electronic configuration	Unpaired electron
$[Fe(CN)_6]^{4-}$	$Fe^{3+} \Rightarrow 3d^5 \Rightarrow t_2g^{2,2,1}, e^{0,0}$	1
$[Fe(CN)_6]^{3-}$	$Fe^{2+} \Rightarrow 3d^6 \Rightarrow t_2g^{2,2,2}, e^{0,0}$	0
$[Ti(CN)_6]^{3-}$	$Ti^{3+} \Rightarrow 3d^1 \Rightarrow t_2g^{1,0,0}, e^{0,0}$	1
$[Ni(CN)_4]^{2-}$	$Co^{3+} \Rightarrow 3d^6 \Rightarrow t_2g^{2,2,2}, e^{0,0}$	0

$[Co(CN)_6]^{3-}$ $Ni^{2+} \Rightarrow 3d^8 \Rightarrow t_2g^{2,2,2},e^{1,1}$ 2

27. (a) $CoCl_3.4\ NH_3$, (b) $CoCl_3.5NH_3$, (C) $CoCl_3.6NH_3$ and (d) $CoCl(NO_3)_2.5NH_3$.

Number of complex(es) which will exist in cis-trans from is / are ____________.

Ans. (1)

Sol. (i) $CoCl_3.4NH_3 \Rightarrow [Co(NH_3)_4Cl_2]Cl$ (ii) $CoCl_3.5NH_3 \Rightarrow [Co(NH_3)_5Cl]Cl_2$

(iii) $CoCl_3.6NH_3 \Rightarrow [Co(NH_3)_6]Cl_3$ (iv) $CoCl(NO_3)_2.5NH_3 \Rightarrow [Co(NH_3)_5Cl](NO_3)_2$

28. The complete combustion of 0.492 g of an organic compound 'C', 'H' and 'O' gives 0.793g of CO_2 and 0.442g of H_2O. The percentage of oxygen composition in the organic compound is _________.

Ans. (46)

Sol. Organic compound + $O_2 \longrightarrow Co_2(g) + H_2O$

[Containing C, H, O] 0.793 gram 0.442 gram

0.492 gram

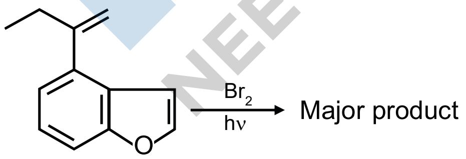
$$W_C = \left[\frac{0.792}{44}\right]12 = 0.216\text{gram}$$

$$W_H = \left[\frac{0.442}{48}\right]2 = 0.491$$

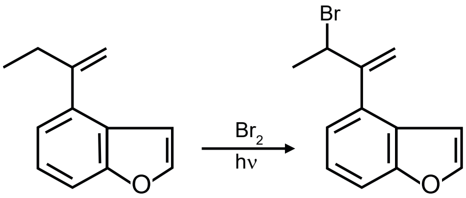
$W_o = [0.492 - 0.2651] = 0.2269$

$$\%\text{of}O = \frac{0.2249}{0.492} \times 100 = 46.11 \approx 46$$

29. The major product of the following reaction contains _________ bromine atom(s).



Ans. (1)

Sol. 

30. 0.01 M $KMnO_4$ solution was added to 20.0 mL of 0.05 Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of $KMnO_4$ solution left in the burette after the end point is _________ mL.

Ans. (30)

Sol. $Fe^{2+} + MnO_4^{-} \longrightarrow Fe^{3+} + Mn^{2+}$

$v_f = 1 \quad v_f = 5$

Mili eq. of mohar's salt = milli eq. of $KMnO_4$

$1 \times [0.05 \times 20] = 5[0.01 \times V_{ml}]$

Volume of $KMnO_4$ left = 30 ml

PART : MATHEMATICS

1. Let $R_1 = \{(a, b) \in N \times N : |a - b| \le 13\}$ and

$R_2 = \{(a, b) \in N \times N : |a - b| \ne 13\}$. Then on N:

(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_2 is an equivalence relation but R_1 is not

Ans. (B)

2. Let f(x) be a quadratic polynomial such that f(– 2) + f(3) = 0. If one of the roots of f(x) = 0 is – 1, then the sum of the roots of f(x) = 0 is equal to :

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$ (C) $\frac{13}{3}$ (D) $\frac{14}{3}$

Sol. Let $f(x) = ax^2 + bx + c = a(x - 1)(x - \alpha)$

$f(-2) = a(-1)(-2 - \alpha) = a(2 + \alpha)$

$f(3) = a(4)(3 - \alpha) = 4a(3 - \alpha)$

$f(-2) + f(3) = 0 \Rightarrow a(2 + \alpha + 12 - 4\alpha) = 0$

$$\Rightarrow a \ne 0,\ -3\alpha + 14 = 0 \Rightarrow \alpha = \frac{14}{3}$$

roots are $= -1, \frac{14}{3}$

some of roots $= -1 + \frac{14}{3} = \frac{11}{3}$

3. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to :

(A) 205 (B) 615 (C) 510 (D) 430

Ans. (D)

4. The term independent of x in the expansion $\left(1 - x^2 + 3x^3\right)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}, x \ne 0$ is :

(A) $\frac{7}{40}$

(B) $\frac{33}{200}$

(C) $\frac{39}{200}$

(D) $\frac{11}{50}$

Ans. (B)

Sol. $\left(1-x^2+3x^3\right)\left({}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r\right.$

$\left(1-x^2+3x^3\right)\left({}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r(x)^{33-5r}\right)$

$33 - 5r \neq 0$

$33 - 5r = -2$

$r = 7$

$33 - 5r \neq -3$

Term independent of x is $= -{}^{11}C_7\left(\frac{5}{2}\right)^4\left(\frac{-1}{5}\right)^7$

$= \frac{11\times10\times9\times8}{24}\times\frac{5^4}{16}\times\frac{1}{5^7}$

$= \frac{33}{200}$

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is :

(A) 21 (B) 22 (C) 23 (D) 24

Ans. (C)

Sol. If d is common difference then 100 = a + (n + 1)d

$d = \frac{100-1}{n+1}$

$\frac{A_1}{A_n} = \frac{a+d}{100-d} = \frac{1}{7}$

$\Rightarrow \frac{a+\frac{100-a}{n+1}}{100-\frac{100-a}{n+1}} = \frac{1}{7}$

$\Rightarrow \frac{an+100}{100n+a} = \frac{1}{7}$

$\Rightarrow$ 7an + 700 = 100n + a
$\Rightarrow$ 7 (33 – n)n + 700 = 100n + 33 – n
$\Rightarrow$ 7n2 – 132n – 667 = 0
$\Rightarrow$ n = 23

6. Let f, g : R $\to$ R be functions defined by

$$f(x)\begin{cases}[x] & , \ x<0 \\ |1-x| & , \ x\geq 0\end{cases} \text{and} \, g(x)=\begin{cases}e^x & , \ x<0 \\ (x-1)^2-1 & , \ x\geq 0\end{cases}$$

Where [x] denote the greatest integer less then or equal to x. Then, the function fog is discontinuous at exactly:

(A) one point (B) two points (C) three points (D) four points

Ans. (B)

7. Let f : R $\to$ R be a differentiable function such that $f\left(\frac{\pi}{4}\right)=\sqrt{2}, f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$ and let $\int_x^{\pi/4}(f'(t)\sec t + \tan t \sec t f(t))\,dt$ for $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right)$. Then $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} g(x)$ is equal to :

(A) 2

(B) 3

(C) 4

(D) – 3

Ans. (B)

Sol. $g(x)=\int_x^{\frac{\pi}{4}} d(f(t).\sec t)=(f(t).\sec t)_x^{\frac{\pi}{4}}$

$$=f\left(\frac{\pi}{4}\right).\sec\frac{\pi}{4}-f(x).\sec x$$

$$=2-\frac{f(x)}{\cos x}$$

$$\lim_{x\to\frac{\pi}{2}^-} g(x)=\lim_{x\to\frac{\pi}{2}^-}\left(2-\frac{f(x)}{\cos x}\right)=2-\lim_{x\to\frac{\pi}{2}^-}\frac{f(x)}{\cos x}\to\frac{0}{0}\text{form}$$

$$=2-\lim_{x\to\frac{\pi}{2}^-}\frac{f(x)}{-\sin x}=2\frac{f'\left(\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}}$$

= 2 + 1 = 3

8. Let $f : R \to R$ be a continuous function satisfying $f(x) + f(x + k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then

(A) $I_1 + 2I_2 = 4nk$

(B) $I_1 + 2I_2 = 2nk$

(C) $I_1 + nI_2 = 4n^2k$

(D) $I_1 + nI_2 = 6n^2k$

Ans. (C)

Sol. $f(x) + f(x + k) = n$ ...(1)

put $x \to x + k$

$f(x + k) + f(x + 2k) = n$ ...(2)

subtract $f(x) – f(x + 2k) = 0$

period is 2k

$$\text{Now, } I_1 = \int_0^{4nk} f(x)dx \text{ b}$$

$$= 2n\int_0^{2nk} f(x)dx$$

$$I_2 = \int_{-k}^{3k} f(x)dx = 2\int_0^{2k} f(x)dx$$

$$I_1 + I_2 = (2n+2)\int_0^{2k} f(x)dx$$

$$= (2n+2)\left[\int_0^{2k} f(x)dx + \int_k^{2k} f(x)dx\right]$$

$$= (2n+2)\left[\int_0^{k} f(x)dx + \int_0^{k} f(x+k)dx\right]$$

$$= (2n+2)\left[\int_0^{k} f(x) + f(x+k)dx\right]$$

$= (2n + 2)\ nk$

Similarly for $I_1 + 2I_2 = (2n + 4)\ nk$

$I_1 + nI_2 = 4n^2k$

9. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is :

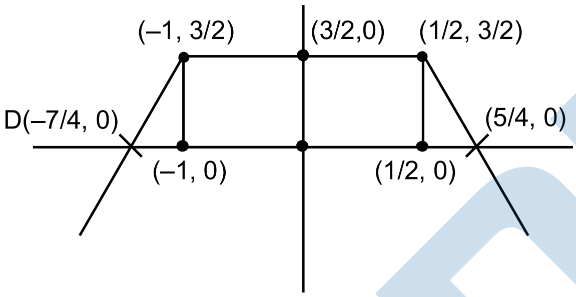
(A) $\frac{9}{4}$ (B) $\frac{45}{16}$ (C) $\frac{27}{8}$ (D) $\frac{63}{16}$

Ans. (C)

Sol. $y = 3 - |x+1| - \left|x - \frac{1}{2}\right|$

$$y=\begin{cases} 3x+x+1+x-\frac{1}{2} & x>-1 \\ 3-x-1+x-\frac{1}{2} & -1\le x\le \frac{1}{2} \\ 3-x-1-x+\frac{1}{2} & x\ge \frac{1}{2} \end{cases}$$

$$y=\begin{cases} 2x+\frac{7}{2} & x<-1 \\ \frac{3}{2} & -1\le x\le \frac{1}{2} \\ -2x+\frac{5}{2} & x\ge \frac{1}{2} \end{cases}$$



$$\text{Requiredarea}=\frac{3}{2}\times\frac{3}{2}+\frac{1}{2}\left(\frac{3}{2}\times\frac{3}{4}\right)=\frac{1}{2}\left(\frac{3}{2}\times\frac{3}{4}\right)$$

$$=\frac{9}{4}+\frac{19}{28}+\frac{19}{28}$$

$$=\frac{9}{4}+\frac{9}{10}+\frac{9}{16}=\frac{27}{8}$$

10. Let x = (y) be the solution of the differential equation $2ye^{x/y^2}dx+\left(y^2-4xe^{x/y^2}\right)dy=0$ such that x(1) = 0. Then, x(e) is equal to :

(A) e $\log_e(2)$

(B) ⁻e $\log_e(2)$

(C) $e^2\log_e(2)$

(D) ⁻$e^2\log_e(2)$

Ans. (D)

11. Let the slope of the tangent to a curve y = f(x) at (x, y) be given by 2 tenx(cosx – y). If the curve passes through the point $\left(\frac{\pi}{4},0\right)$, then the value of $\int_0^{\pi/2} ydx$ is equal to:

(A) $(2-\sqrt{2})+\frac{\pi}{\sqrt{2}}$

(B) $2-\frac{\pi}{\sqrt{2}}$

(C) $(2+\sqrt{2})+\frac{\pi}{\sqrt{2}}$

(D) $2+\frac{\pi}{\sqrt{2}}$

Ans. (B)

Sol. Slope of tangent $\Rightarrow \frac{dy}{dx}=2\tan x(\cos x-y)$

$\Rightarrow \frac{dy}{dx}=2\tan x.y=2\sin x$

I.F. = $e^{\int 2\tan xdx}$ = $e^{2\ell n\sec x}$ = $e^{\ell n\sec^2 x}$ = $\sec^2 x$

Solution of equation

$y.\sec^2 x=\int \sec^2 x.2\sin xdx+C$

$\Rightarrow y\sec^2 x=2\int \sec x\tan xdx+C$

$\Rightarrow y\sec^2 x=2\sec x+C$

$\because$ curve passes through $\left(\frac{\pi}{4},0\right)$

0 = 2secπ/4 + C

$C=-2\sqrt{2}$

$\Rightarrow$ curve $y\sec^2 x=2\sec x-2\sqrt{2}$

$\Rightarrow y=2\cos x-2\sqrt{2}\cos^2 x2\cos x-\sqrt{2}(1+\cos 2x)$

$\int_0^{\pi/2} f(x)dx=\int_0^{\pi/2}\left(2\cos x-\sqrt{2}-\sqrt{2}\cos 2x\right)dx\left(2\sin x-\sqrt{2}x-\frac{\sin 2x}{\sqrt{2}}\right)_0^{\pi/2}$

$\left(2(1)-\sqrt{2}.\frac{\pi}{2}-0\right)-0(0-0-0)=2-\frac{\pi}{\sqrt{2}}$

12. Let a triangle be bounded by the lines L_1 : 2x + 5y = 10; L_2: – 4x + 3y = 12 and the lines L_3, which passes through the point P (2, 3), intersects L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the tringle is equal to:

(A) $\frac{110}{13}$ (B) $\frac{132}{13}$ (C) $\frac{142}{13}$ (D) $\frac{151}{13}$

Ans. (B)

13. Let a > 0, b > 0. Let and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14} l \text{ and } (e')^2 = \frac{11}{8} l'$, then the value of 77a + 44b is equal to :

(A) 100

(B) 110

(C) 120

(D) 130

Ans. (D)

Sol. $e^2 = \frac{11}{14}\ell \Rightarrow 1 + \frac{b^2}{a^2} = \frac{11}{14} \cdot \frac{2b^2}{2}$

$= a^2 + b^2 = \frac{11 b^2 . a}{7}$

$\Rightarrow 7a^2 + 7b^2 = 11ab^2$

$\because (e')^2 = \frac{11}{8}\ell' \Rightarrow 1 + \frac{a^2}{b^2} = \frac{11}{8} \cdot \frac{2a^2}{b}$

$\Rightarrow a^2 + b^2 \frac{11}{4} a^2 \mathbf{b}$

$\Rightarrow 4a^2 + 4\ b^2 = 11a^2b$

equation (1) and (2)

$\frac{7}{4} = \frac{b}{a}$

$\because 1 + \frac{b^2}{a^2} = \frac{11}{7}\frac{b^2}{a^2} \Rightarrow 1 + \frac{49}{16} = \frac{11}{7} \times \frac{7}{4} \times b$

$\Rightarrow b \frac{65}{44} \Rightarrow 44b = 65$

$\because 1 + \frac{a^2}{b^2} = 1 + \frac{16}{49} = \frac{11}{4} \times \frac{4}{7} \times a$

$\Rightarrow 65 = 77a$

77a + 44b = 130

14. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} - \hat{k}$, where $\alpha \in$ R. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a}.\vec{b})|\vec{b}|^2$ is equal to:

(A) 10

(B) 7

(C) 9

(D) 14

Ans. (D)

15. If vertex of a parabola is (2, –1) and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is:

(A) 2 (B) 8 (C) 12 (D) 16

Ans. (B)

Sol. Distance between directrix and vertex is $a = \left|\frac{8+3-21}{5}\right| = 2$

Now length of latus rectum = 4a = 8

16. Let the plane $ax + by + cz = d$ pass through (2, 3, – 5) and is perpendicular to the planes

$2x + y - 5z = 10$ and

$3x + 5y - 7z = 12$.

If a, b, c, d are integers $d > 0$ and gcd (|a|, |b|, |c|, d) = 1, then the value of $a + 7b + c + 20d$ is equal to:

(A) 18 (B) 20 (C) 24 (D) 22

Ans. (D)

17. The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

(A) $\frac{1}{24}$

(B) $\frac{1}{40}$

(C) $\frac{1}{30}$

(D) $\frac{1}{20}$

Ans. (D)

18. The value of $\lim_{x\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to:

(A) 1

(B) 2

(C) 3

(D) 6

Ans. (C)

Sol. $\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2+3r+3}\right) = \sum_{r=1}^{n} \tan^{-1}\left(\frac{(r+2)-(r+1)}{1+(r+1)(r+2)}\right)$

$\sum_{r=1}^{n} \tan^{-1}\left(\tan^{-1}(r+2) - \tan^{-1}(r+1)\right)$

$= (\tan^{-1}(3) - \tan^{-1}(2)) + (\tan^{-1}(4) - \tan^{-1}(3)) + \text{-------} + (\tan^{-1}(n+2) - \tan^{-1}(n+1))$

$= \tan^{-1}(n+2) - \tan^{-1}(2) = \tan^{-1}\left(\frac{(n+2)-2}{1+2(n+2)}\right)$

$= \tan^{-1}\left(\frac{n}{2n+5}\right)$

$\lim_{n\to\infty} 6\tan\left(\tan^{-1}\frac{n}{2n+5}\right) = \lim_{n\to\infty} \frac{6n}{2n+5} = 6 \times \frac{1}{2} = 3$

19. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$,then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $\frac{1}{3}$

(B) 1

(C) $\frac{5}{3}$

(D) $\frac{7}{3}$

Ans. (C)

20. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, the value of tan $(\alpha + \beta)$ and quadrant in which $\alpha + \beta$ lies, respectively are :

(A) $-\frac{1}{7}$ and IV[th] quadrant

(B) 7 and I[st] quadrant

(C) – 7 and IV[th] quadrant

(D) $\frac{1}{7}$ and I[st] quadrant

Ans. (A)

Sol. $\cot\alpha = 1 \Rightarrow \tan\alpha = 1$

$$\sec\beta = \frac{-5}{3} \Rightarrow \tan\beta = \frac{-4}{3}$$

$$\text{Now } \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{1-\frac{4}{3}}{1-1\times\left(\frac{-4}{3}\right)} = \frac{-1}{7}$$

$$\text{also } \pi < \alpha < \frac{3\pi}{2}$$

$$\frac{\pi}{2} < \beta < \pi$$

$$\frac{3\pi}{2} < \alpha+\beta < \frac{5\pi}{2}$$

Since tan ($\alpha + \beta$) is negative so $\alpha + \beta$ lies in IV quadrant

21. Let the image of the point P(1,2, 3) in the line L : $\frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. Let R (α, β, γ) be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of 22 ($\alpha + \beta + \gamma$) is equal to ___________.

Ans. (125)

22. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less then 50 marks, then in worst case, the number of students can fail is_________.

Ans. (0)

23. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $\left(x-2\sqrt{2}\right)^2 + \left(y-\sqrt{2}\right)^2 r^2$, then value of r^2 is equal to _________.

Ans. (10)

24. If $\lim_{x\to 1} \frac{\sin\left(3x^2-4x+1\right) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of (a – b) is equal to ________.

Ans. (11)

25. Let for n = 1, 2, ….., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$ is equal to _________.

Ans. (41651)

26. If the system of linear equations

$2x - 3y = \gamma + 5,$

$\alpha x + 5y = \beta = 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value $|9\alpha + 3\beta + 5\gamma|$ is equal to ________.

Ans. (58)

27. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set {$n \in$ {1, 2, ……, 100} : $A^n = A$} is _________.

Ans. (25)

28. Sum of squares of modulus of all the complex number z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to

Ans. (2)

29. Let S = {1, 2, 3, 4}. Then the number of elements in the set {f ; $S \times S \to S$: f is onto and f(a, b) =f (b, a) $\geq$ a $\forall$ (a, b) $\in$ s × s} is __________.

Ans. (37)

30. The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee r \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by and assignment of the values p, q, r and s, is equal to _________.

Ans. (9)